COLLIDING SPHERICALLY SYMMETRIC NULL DUST IN EQUILIBRIUM

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We present two recently obtained solutions of the Einstein equations with spherical symmetry and one additional Killing vector, describing colliding null dust streams.

1 Introduction

Null dust is a model for the incoherent superposition of waves with random polarizations and phases, moving in a single direction with the speed of light. It also describes the high frequency (geometrical optics) approximation for any type (including gravitational) radiation. Null dust is characterized by the energy-momentum tensor $T^{ab} = \rho l^a l^b$. Neither the energy density ρ nor the null vector l are uniquely determined and the former can be absorbed in the latter by an appropriate rescaling. Various exact solutions of the Einstein equations in the presence of null dust are known [1,2,3], but certainly the most famous of them is Vaidya's shining star solution [4].

The energy-momentum tensor describing the collision of two streams of null dust (with propagation vectors k and l pointing in opposite spatial directions) is a sum of the two single null dust energy-momentum terms:

$$T^{ab} = l^a l^b + k^a k^b \ . \tag{1}$$

The Einstein equations for the matter source (1) in the plane symmetric case were solved by Letelier [5]. The cylindrically symmetic case was studied by Letelier and Wang [6]. Progress in the numerical treatment of the spherically symmetric case was achieved by Holvorcem, Letelier and Wang [7]. Under the assumption of staticity Date [8] integrated half of the relevant equations. The solution was found by Kramer [9] and by us [10], ours containing two additional parameters. Later, assuming homogeneity in addition to spherical symmetry, we have obtained a Kantowski-Sachs type solution: a closed Universe filled with colliding streams of radiation [11]. The interpretation of such spherically symmetric solutions as regions of space-time containing both escaping and backscattered gravitational radiation was proposed by Poisson and Israel [12].

Recently Kramer obtained static and stationary solutions describing the collision of cylindrical null dust beams [13].

Here we present briefly the spherically symmetric static and homogeneous solutions.

2 The spherically symmetric static and homogeneous solutions

The detailed derivation of these solutions was presented elsewhere [10,11]. The line element including both the homogeneous case ($c = 1$) and the static case ($c = -1$) can be given in the concise form:

$$ds^2 = \frac{e^{L^2}}{cCR}\left(dZ^2 - 2R^2 dL^2\right) + R^2 d\Omega^2 \ . \tag{2}$$

Here $R(L)$ is a transcendental function (depending on the error function):

$$cCR = e^{L^2} - 2L\Phi_B(L), \qquad \Phi_B(L) = B + \int^L e^{x^2} dx > 0 \ , \tag{3}$$

while B and $C > 0$ are constants of integration.

One may think of Z as being a time coordinate t in the static case and a radial coordinate r in the homogeneous case. Conversely, the coordinate L has the meaning of time in the homogeneous case and is a radial coordinate in the static case. However, for this interpretation to hold, a study of the sign of the expression (3) of cCR is necessary, more precisely in both cases $R \geq 0$ should hold. This requirement follows also from the energy conditions [10,11].

First of all let us remark that there is a true singularity at $R = 0$. This can be seen for example from the ill-behaveness at $R = 0$ of the Kretschmann scalar $\mathcal{R}_{abcd}\mathcal{R}^{abcd} \propto R^{-6}$. Thus the domain of the coordinate L is bounded by its value(s) L_0 at $R = 0$. The constant B is related to L_0 as:

$$B = \chi(L_0) = \frac{e^{L_0^2}}{2L_0} - \int^{L_0} e^{x^2} dx \ . \tag{4}$$

For each value of the constant B there are two values $L_{0\pm}$, a positive and a negative one (Fig.1). These are situated symmetrically with respect to the origin only for $B = 0$.

Second, we express the function cCR in the alternative form

$$cCR = 2L\left[\chi(L) - \chi(L_0)\right] \ . \tag{5}$$

Because $\chi(L)$ is monotonically decreasing: $d\chi/dL = -e^{L^2}/2L^2 < 0$, the domains of positivity of R are $L_{0-} < L < L_{0+}$ in the homogeneous case and $L < L_{0-}$ together with $L > L_{0+}$ in the static case.

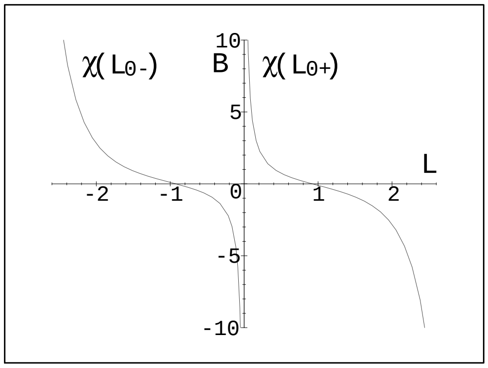

Figure 1: The curves $B = \chi(L_{0\pm})$, corresponding to the singularities $R = 0$, devide the horizontal axis L into domains where $R > 0,\ < 0,\ > 0$ (static case) or $R < 0,\ > 0,\ < 0$ (homogeneous case).

3 The interpretation of the solutions

In the homogeneous case the time coordinate L is bounded both from above and below, the boundaries being singularities. The radius function R is increasing for $L \in (L_{0-}, L^*)$, then decreasing for $L \in (L^*, L_{0+})$, where L^* is defined by $\Phi_B(L^*) = 0$. The solution represents a closed Universe [11]. From the symmetry group it is obvious that this is a Kantowski-Sachs type solution, difficult to relate to any physical situation.

We have analyzed in detail the static solution [10] for $L > L_{0+}$. The metric is not asymptotically flat and close to the axis $R = 0$ (which is the singularity) its mass decreases to negative values. Thus we run again into interpretational difficulties. However, cutting the interior region along a timelike hypersurface, we can glue the solution to some physically meaningful matter. We have discussed the junction with a generic static interior [10] by applying the Darmois-Israel [14,15] matching procedure. In particular the junction algorithm with the interior Schwarzschild solution [16] has fixed the free parameters B and C of the colliding null dust. The natural exteriors are the incoming and outgoing Vaidya regions, which were matched [10] along null hypersurfaces to our solution, by applying the Barrabès-Israel junction procedure [17].

In the range $L < L_{0-}$ we have just an other copy of the static solution with parameters $-B$ and C, and $-L$ as the radial variable. Indeed, the metric (2) is invariant under the interchange $(L, B) \rightarrow (-L, -B)$.

These solutions can be reinterpreted as anisotropic fluids with radial pres-

sures equal with their energy densities, and no pressures in tangential directions to the spheres. An other approach we already presented elsewhere [11] is based on the connections with two-dimenssional dilatonic models. In this context Miković [18] has given a solution in the form of a perturbative series in powers of the outgoing part of the energy-momentum tensor.

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